**Homework6**



**P16.1.22** (a) Derive the FSE of the waveform of Figure P16.1.22 by direct evaluation. (b) Show that if it is added to a delayed and negated version, the result agrees with Equation 16.3.19. (c) Indicate how the FSE can be obtained as the product of a rectangular pulse train of unit height (Figure 16.2.5 and a triangular waveform derived from that of Figure 16.3.4.

**Solution:** (a) The function is even, and *a*0 = *C*0= ; =

=. Hence, .

(b) When the function is negated and either advanced or delayed by *T*/2, the phase angle changes by *nω*0*T*/2 = *nπ*. Adding *nπ*to, or subtracting*nπ*from, a cosine function gives the same result. Thus cos(*α*±*nπ*) =. The function becomes: 2π) += . When this is added to *f*(*t*), the dc component and the even harmonics cancel out and the odd harmonics add, giving –*ftr*(*t*) of Equation 16.3.19, with *Am* replaced by *A*.

(c) In principle, the FSE of the required function could be obtained as the product of –*ftr*(*t*) of Equation16.3.19 and the FSE of the rectangular pulse train shown. The latter is from Equation 16.2.24, with *α* = 1/2 and *A* = 1:

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**P16.1.23** (a) Derive the FSE of the waveform of Figure P16.1.23 by direct evaluation. (b) Indicate how the FSE can be obtained as the product of a rectangular pulse train of unit height (Figure16.2.5) and a sawtooth waveform derived from that of Figure16.2.1.



Solution: (a) *a*0 = *C*0= ; = = . Hence, , and . The FSE is therefore: 

(b) The FSE of the sawtooth waveform shown is obtained from Equation 16.2.7 by ignoring the dc term and replacing *A* by 2*A*: . The rectangular pulse train is that used in P16.1.22 advanced by *T*/4, i.e., *π*/2: +. The FSE of the required waveform is the product of these two FSEs.



**P16.1.27** Obtain the FSE of a full-wave rectified waveform in two ways: (a) as the sum of two half-wave rectified waveforms, with one waveform shifted by half a period with respect to the other waveform; (b) as the product of a square wave of zero average and .

**Solution:**(a) *fhw*(*t*), *n* = 1, 2, 3, …If delayed by *T*/2, the fundamental becomes . The *n*th term becomes = . Adding the two functions gives Equation 16.4.4.

(b) If *A*cos*ω*0*t* is multiplied by a square wave unit amplitude, zero average, centered at the origin, a full-wave rectified waveform is obtained. From Eq. 16.2.25, the FSE of the square waveform is: . Multiplying by *A*cos*ω*0*t* gives: *ffw*(*t*) = =  + =

+ , which is the same as Equation16.4.4.

**P16.2.9** *vSRC*in Figure P16.2.9is the full-wave rectified waveform of Figure16.4.1b having *T* = 1/50 s and an amplitude of 50 V. The purpose of the *LC* filter is to attenuate the ac components of *vSRC*, leaving a near-dc voltage across the 4 kΩload. Determine the first four nonzero terms in the FSE of  and compare with those of *vSRC*.



**Solution:***ω*o== 100*π*. From Equation16.4.4,*vSRC*=+  V retaining the first four terms. In terms of phasors: . The dc term remains unchanged. At *ω*= 200*π*, the first ac term is . Its magnitude is = 3 V and its phase angle is -tan-1= 12.8°. At *ω*= 400*π*, the amplitude of the second ac term is = 0.138 V and its phase angle is -180°= -174.1°. At *ω*= 600*π*, the amplitude of the third ac term is = 0.026 V and its phase angle is 3.85°. The first four terms of the FSE are: *v*O(*t*) = 31.83 + 3.00cos(200*πt* + 12.8°) +

0.138cos(400*πt* -174.1°) + 0.026cos(600*πt* +3.85°) V. Relative rates of attenuation are 1 for the dc term = 0.14 for the fundamental, = 0.033 for the second harmonic, and = 0.014 for the third harmonic. rms of ac components of output is = 2.12 V.

**P16.3.23** Determine the rms value of the voltage shown in Figure 16.3.23 over the time interval (0, 5) s.



**Solution:** For 0 <*t*< 1, *v*(*t*) = *t*, *v*2(*t*) = *t*2, and area under the square is . For 1 <*t*< 3, the area under the square is 1×2 = 2. The area under the square for 3 <*t*< 4 is the same as . The total area under the square is 2 + 5/3 = 11/3. The mean is 11/15, and the rms value is .



**P16.3.26** The periodic voltage *vSRC* is applied as shown in Figure P16.3.26. Determine the average power dissipated in the circuit.

**Solution:** *vSRC* has a dc component 10×1/2 = 5 V. The dc component dissipatespower in the 10 Ω resistor only, amounting to 25/10 = 2.5 W. The rms of the ac component of the triangular waveform is  V. This is applied to two 10 Ω resistors in parallel, that is 5 Ω. The power dissipated is  W. The total power is  W.

**P17.1.5** Two impedances 9.8∠-78°Ω and 18.5∠21.8°Ω are connected in parallel and the combination is connected in series with an impedance 5∠53°Ω. If the circuit is connected across a 100 V rms source, determine the real power delivered by the source.

**Solution:** The admittance of 9.8∠-78°Ω = 0.1020∠78° = 0.0212 + *j*0.0998 S. The admittance of 18.5∠21.8°Ω = 0.0502 – *j*0.0201 S. The total admittance is 0.0714 + *j*0.0797 S = 0.107∠48.16° S which is equivalent to 9.34∠-48.16°Ω = 6.23 – 6.96 Ω. The 5∠53°Ω = 3.01 + *j*3.99. The sum of the two impedances is 9.24 – *j*2.97 Ω. The magnitude of this impedance is 9.71 Ω. The magnitude of the current is 100/9.71 = 10.3 A. The real power is (10.3)^2×9.24 = 980.9 W.



**P17.1.6** The capacitor in the circuit of Figure P17.1.6 absorbs -200 VAR. Determine the power dissipated in the 5 Ω resistor.

Solution: ; ; ; power dissipated in the 5 Ω resistor is 400/5 = 80 W.